Fine-grained complexity of Longest Common increasing Subsequence

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joint work with
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Example: LCS(edu**cat**ed, **c**o**a**u**t**hor) $= 3$

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SETH (Strong Exponential Time Hypothesis)

No $O((2-\varepsilon)^N)$ time algorithm for CNF-SAT on N variables

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Question: Can we solve LCIS in $O(n^{2-\varepsilon})$?

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LCIS: Given two **integer** sequences of length n, determine the length of their **longest common increasing subsequence**

Question: Can we solve LCIS in $O(n^{2-\varepsilon})$?
Our answer: **NO** (unless SETH fails)

Proof overview

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Orthogonal Vectors problem (OV)

Input: Two sets of d-dimensional $(0,1)$-vectors $\{u_0, \ldots, u_{n-1}\}, \{v_0, \ldots, v_{n-1}\} \subset \{0,1\}^d$

Output: u_i, v_j such that $u_i \cdot v_j = 0$

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(1) Vector gadgets:

(2) Glue:

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(You'd figure it out in 5 minutes)

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(Hard work happens here)

Vector gadgets

For u_i's:

$$0 \text{ at index } \ell \longrightarrow 2\ell, 2\ell+1$$
$$1 \text{ at index } \ell \longrightarrow 2\ell, 2\ell$$

For v_i's:

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Example:

$$\begin{aligned} u_i = 0\ 1\ 0\ 1 &\longrightarrow U_i = 01\ 22\ 45\ 66 \\ v_j = 0\ 0\ 1\ 1 &\longrightarrow V_j = 10\ 32\ 55\ 77 \end{aligned}$$

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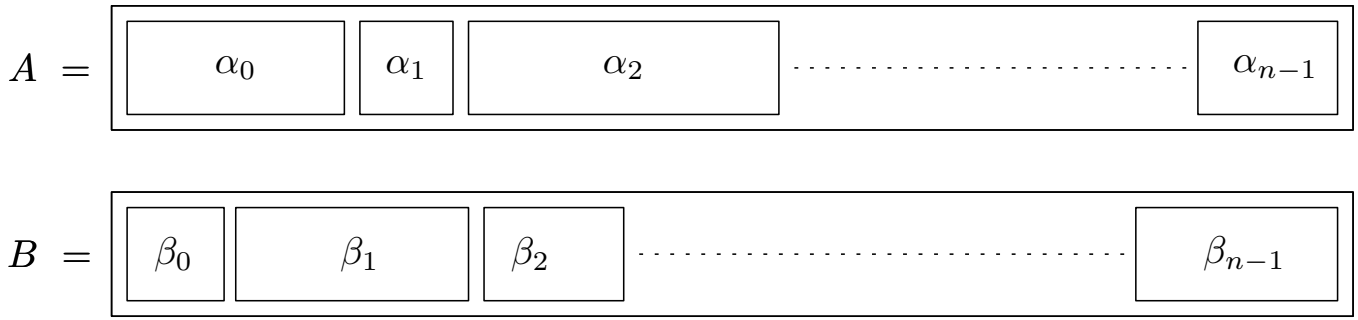
- at most one of 2ℓ and $2\ell+1$ appears in LCIS
- it appears iff $u_i[\ell]$ and $v_j[\ell]$ are not both 1
- $\mathrm{LCIS}(U_i, V_j) = d - (u_i \cdot v_j)$
- $\mathrm{LCIS}(U_i, V_j) \geqslant d$ iff $u_i \cdot v_j = 0$

Glue: separator sequences

$A =$ 

$B =$ 

Glue: separator sequences



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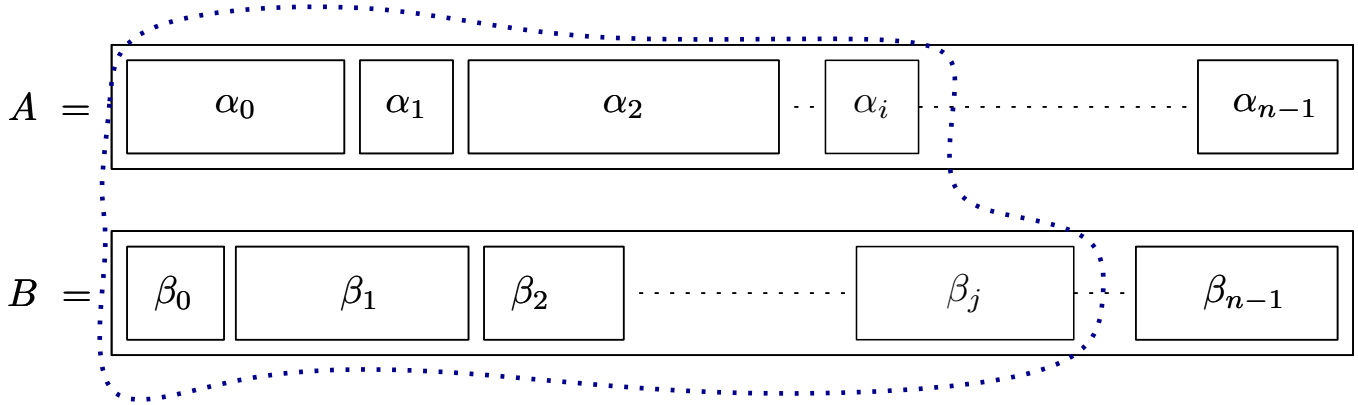

$$\mathrm{LCIS}(\alpha_0 \ldots \alpha_i, \beta_0 \ldots \beta_j) = i + j$$

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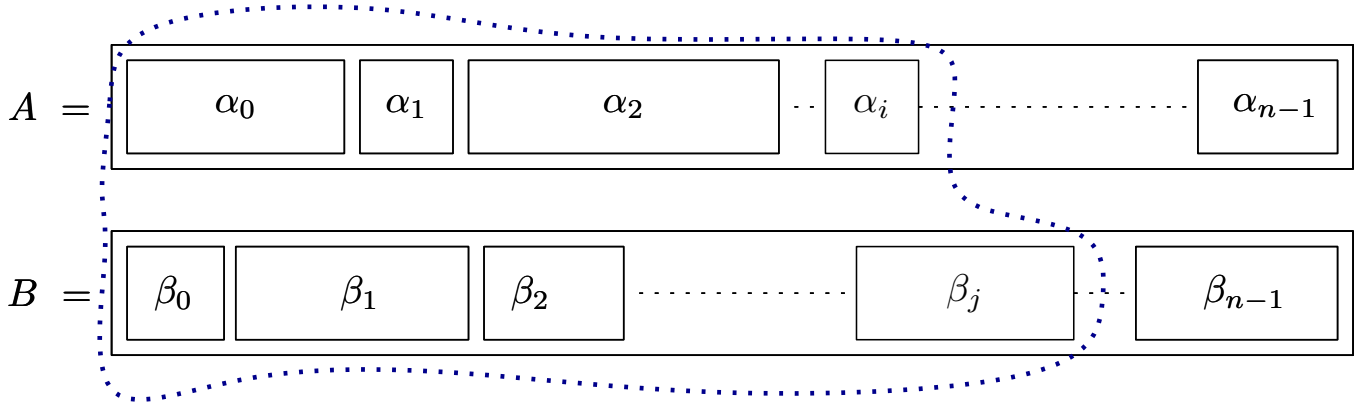

$$\mathrm{LCIS}(\alpha_0 \ldots \alpha_i, \beta_0 \ldots \beta_j) = d \cdot (i + j) + const$$

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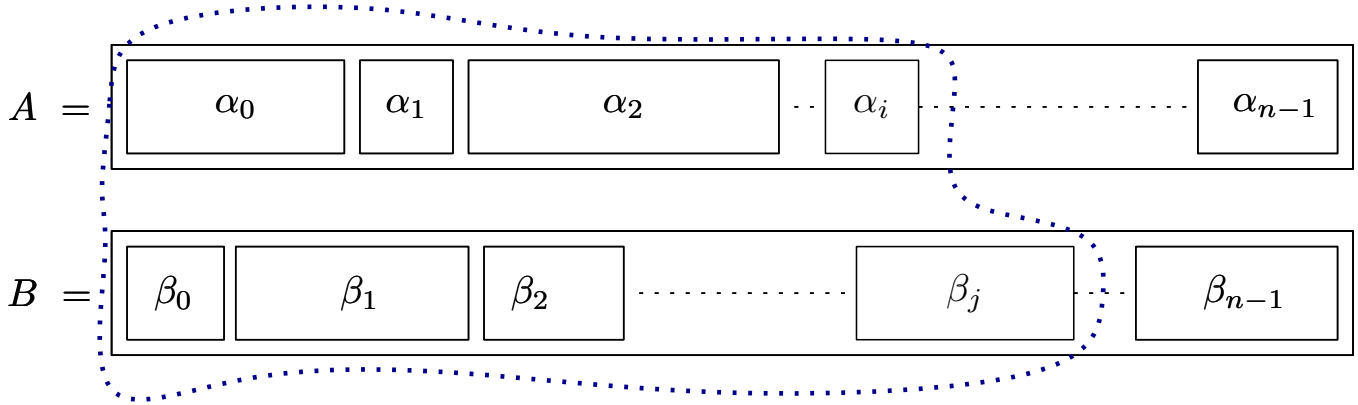

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reverse and negate A, B to obtain $\hat{A}, \hat{B}$

$$\mathrm{LCIS}(\hat{\alpha}_i \ldots \hat{\alpha}_{n-1}, \hat{\beta}_j \ldots \hat{\beta}_{n-1}) = d \cdot (2n - i - j) + const$$

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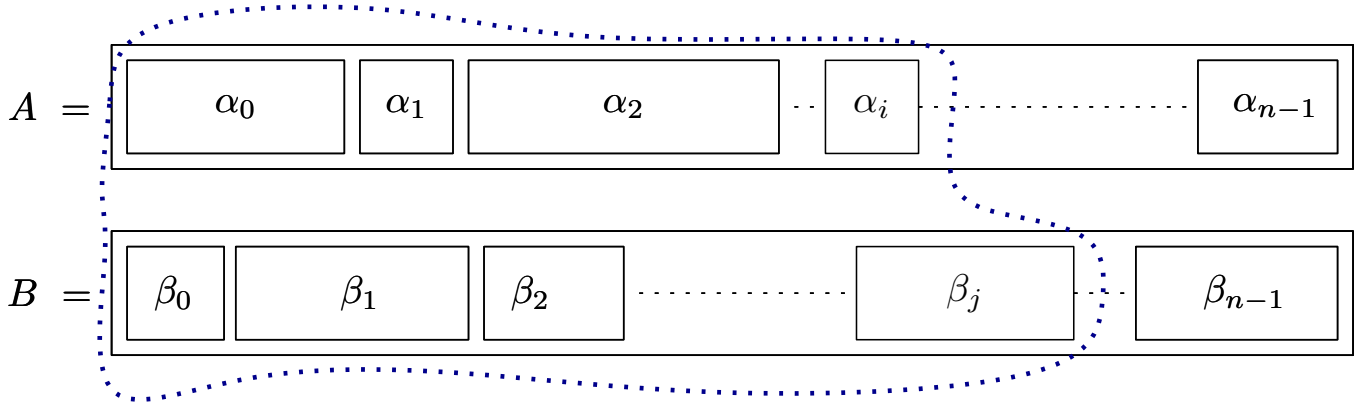

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shift alphabets so that $\Sigma_{A,B} < \Sigma_{U_i,V_j} < \Sigma_{\hat{A},\hat{B}}$

$$X = \alpha_0 U_0 \hat{\alpha}_0 \ \alpha_1 U_1 \hat{\alpha}_1 \ \dots\dots\dots\dots\dots \ \alpha_{n-1} U_{n-1} \hat{\alpha}_{n-1}$$
$$Y = \beta_0 V_0 \hat{\beta}_0 \ \beta_1 V_1 \hat{\beta}_1 \ \dots\dots\dots\dots\dots \ \beta_{n-1} V_{n-1} \hat{\beta}_{n-1}$$

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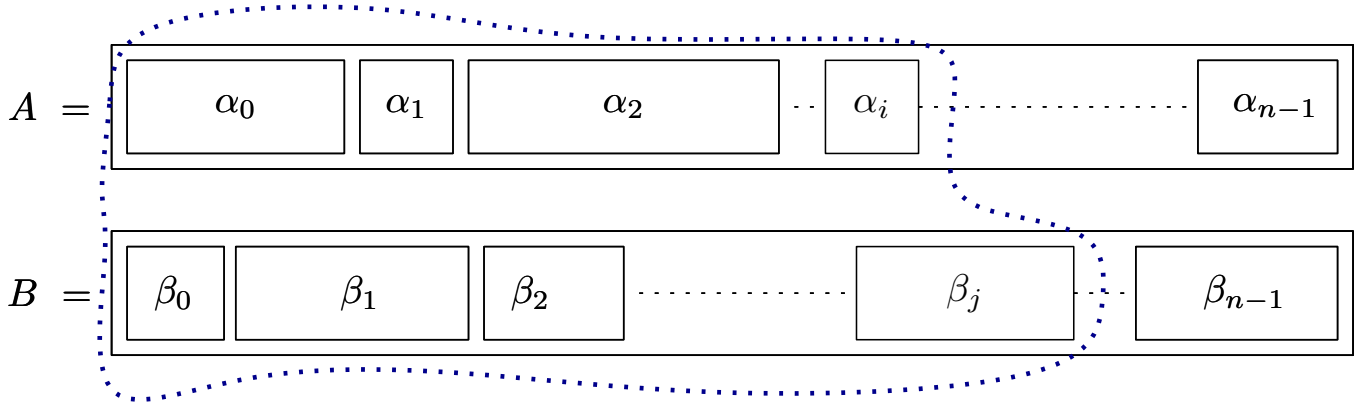

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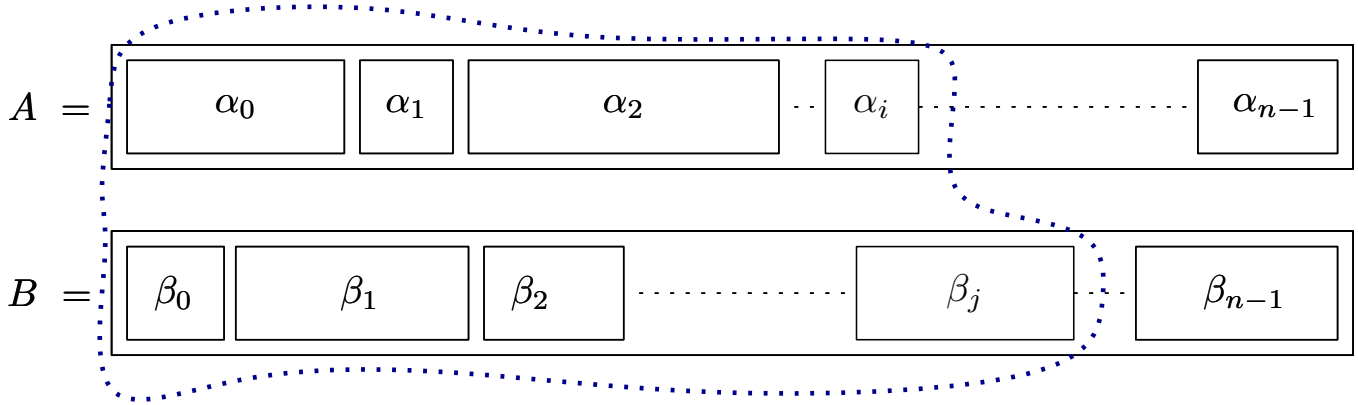
$$\mathrm{LCIS}(X, Y) = const' + \max_{i,j} \mathrm{LCIS}(U_i, V_j) \quad (\text{large iff } \exists_{i,j}\ u_i \cdot v_j = 0)$$

Glue: separator sequences



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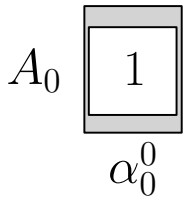
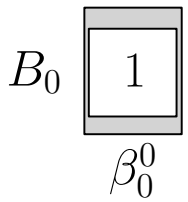
Separator sequences





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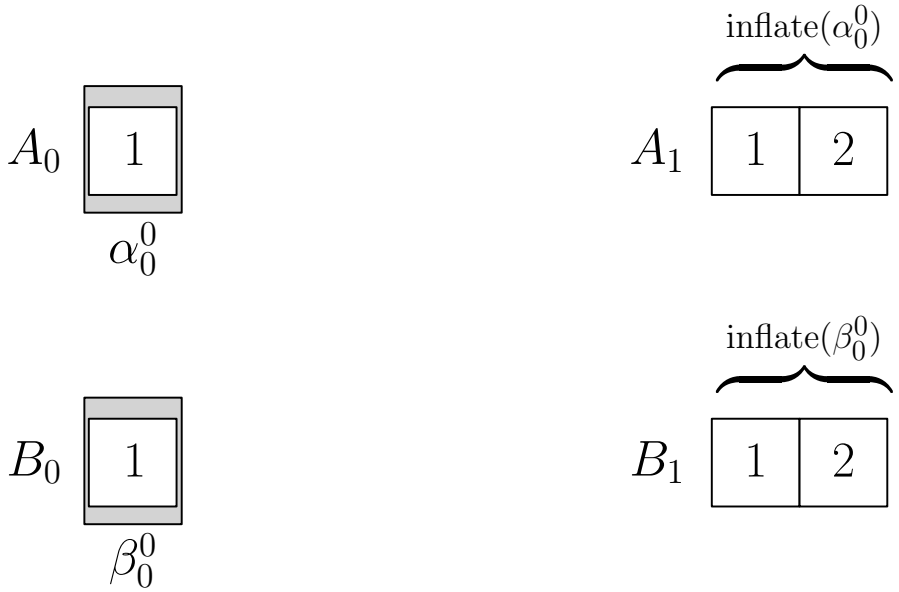

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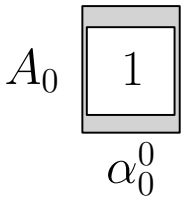
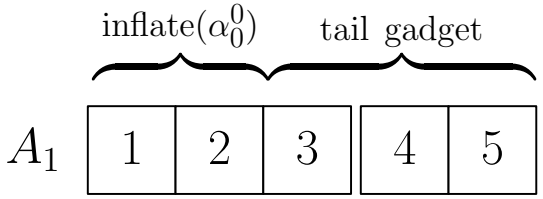
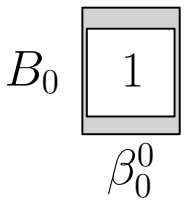
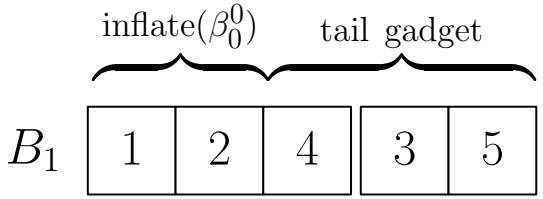
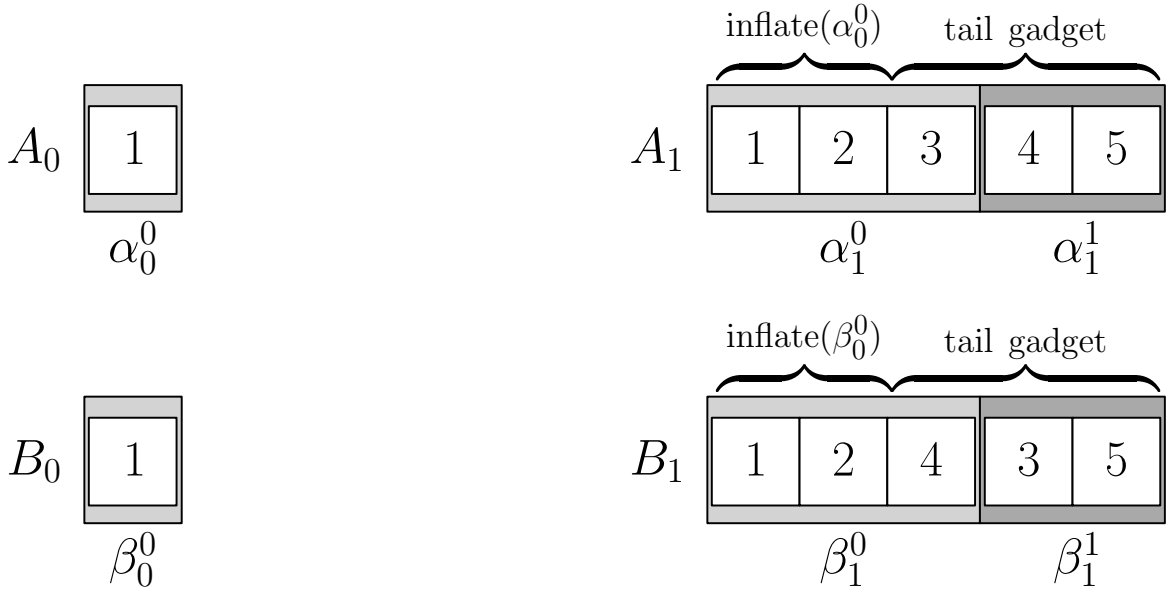








Separator sequences



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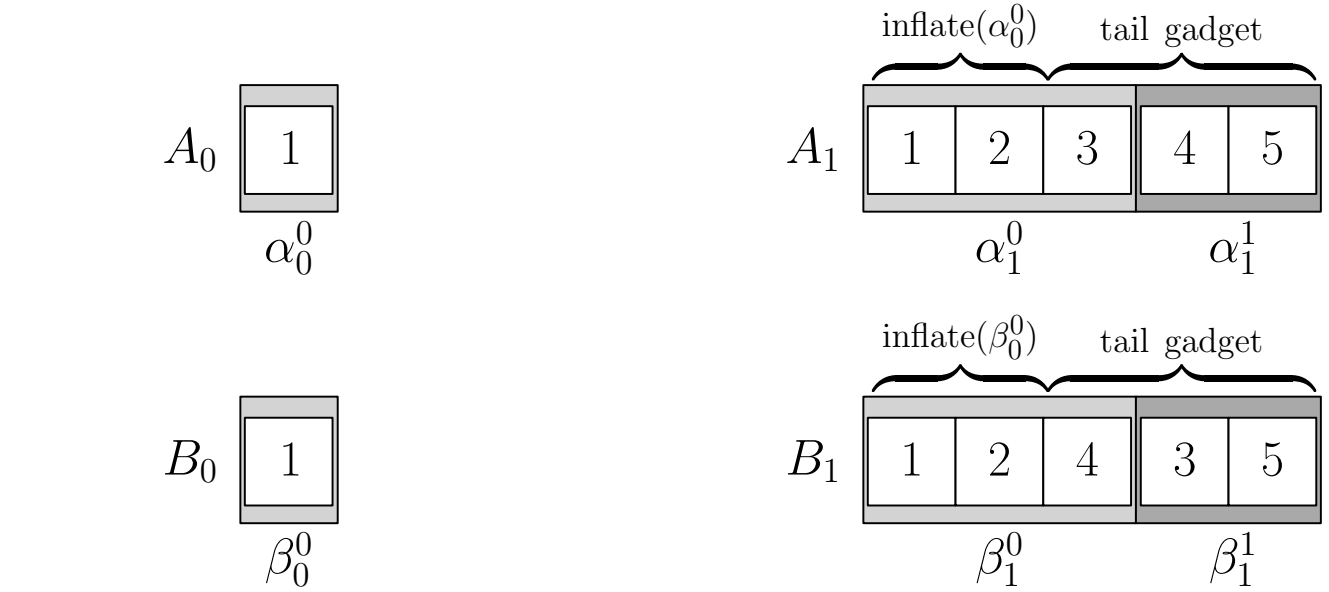

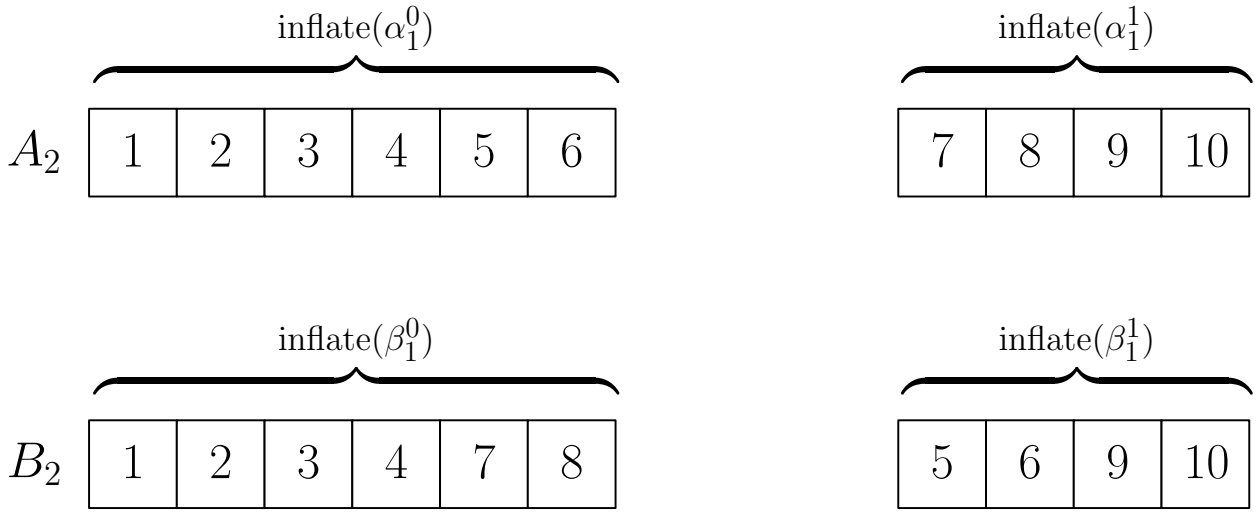

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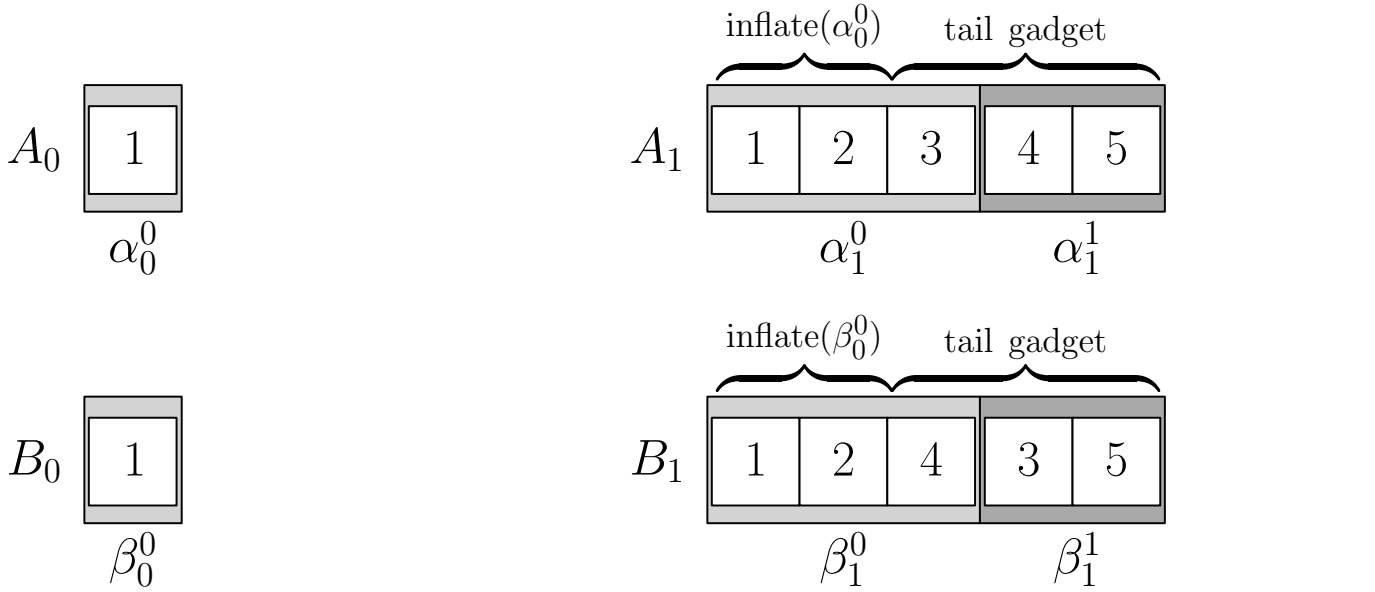

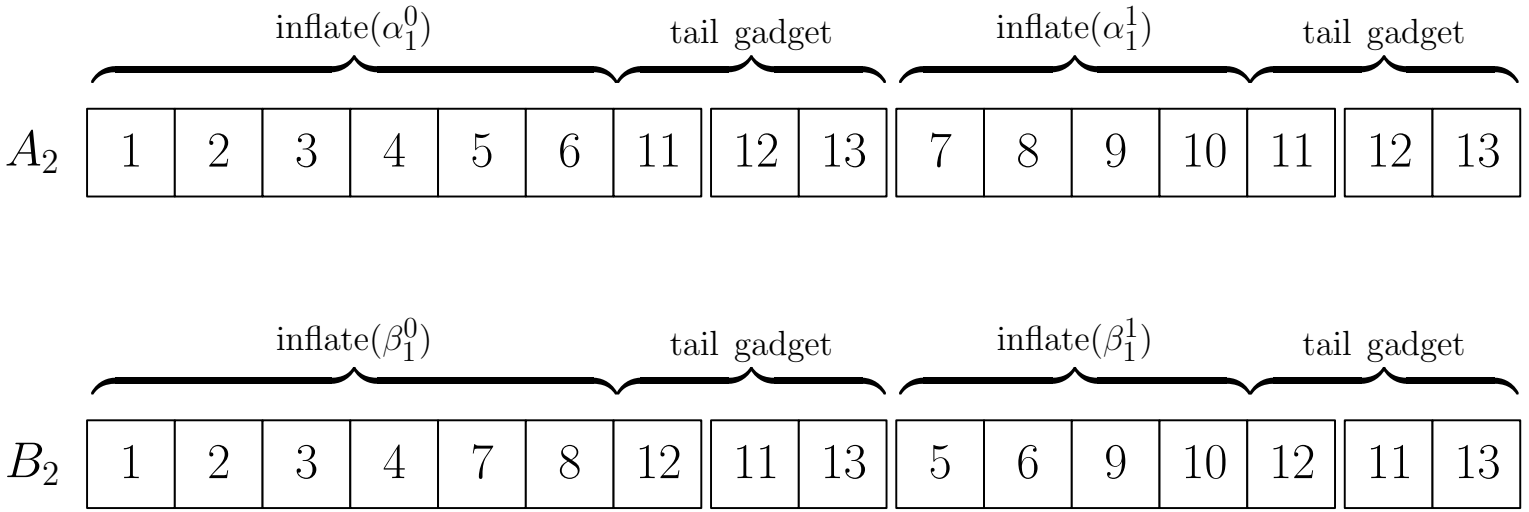

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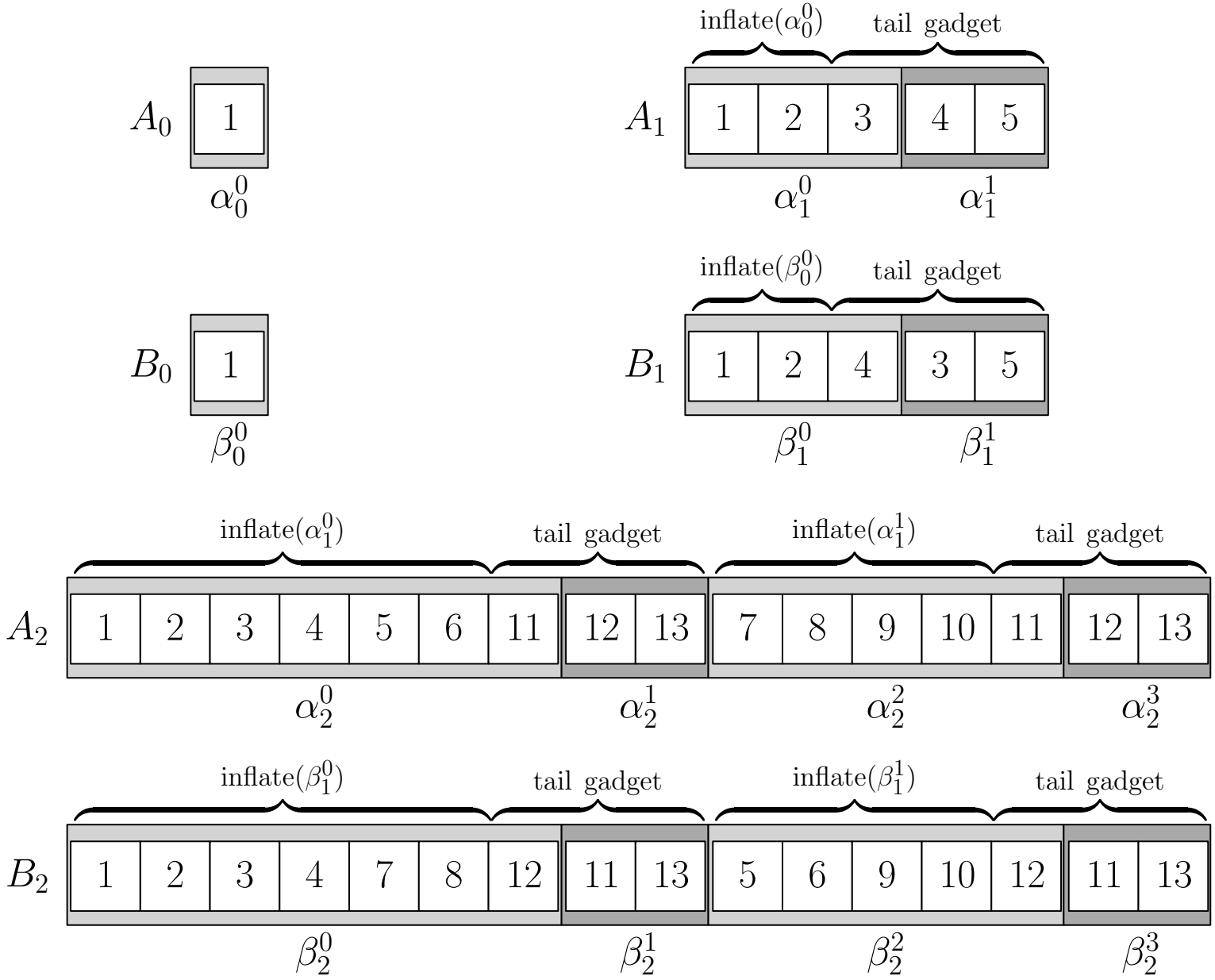

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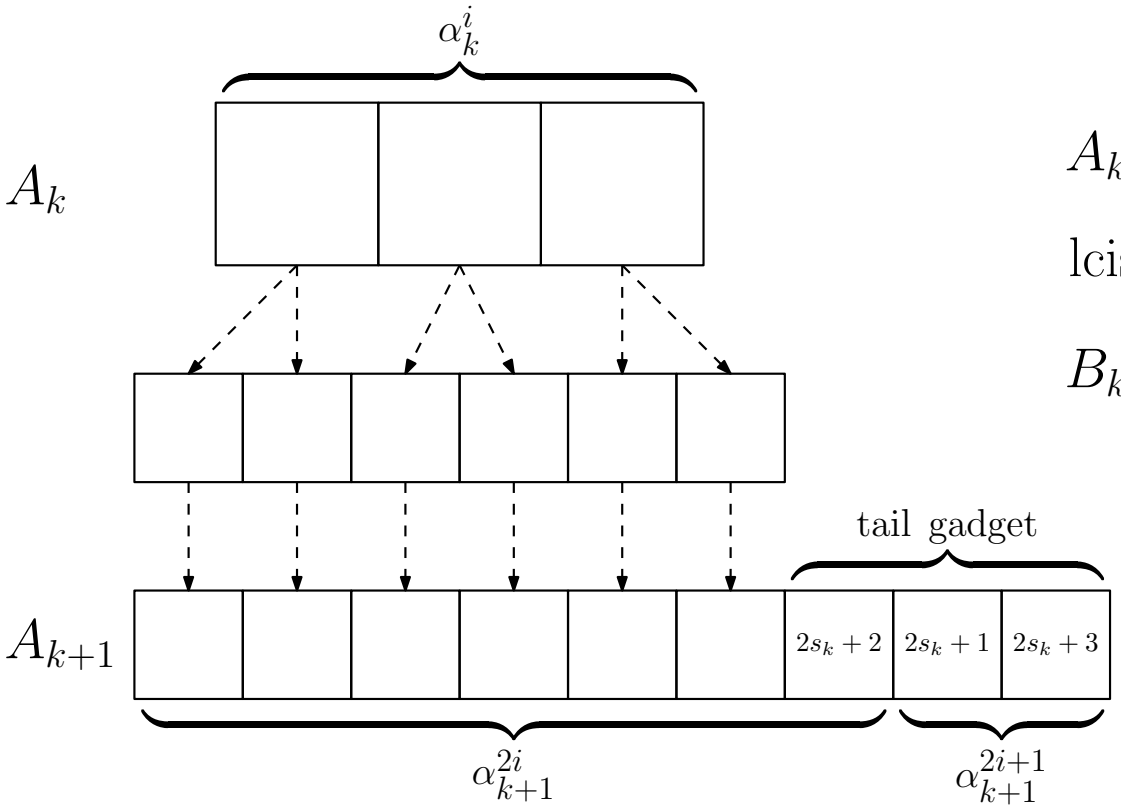

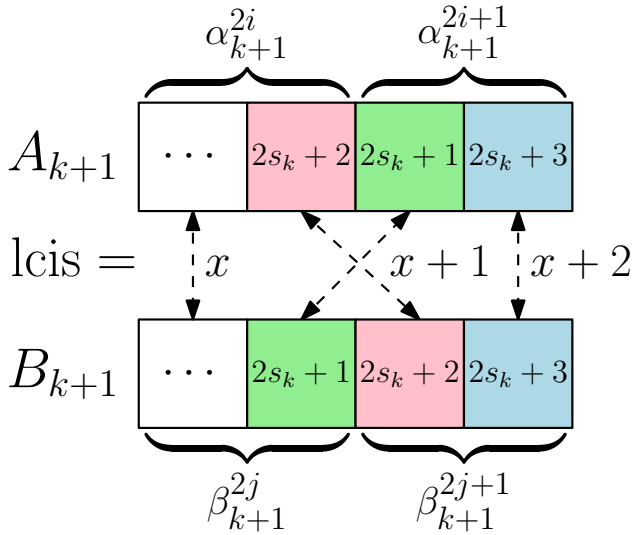

$$x = 2i + 2j + 2^{k+1}$$

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BP-SETH: Brute-force optimal for SAT on **branching programs**, in particular: NC-circuits, $o(\sqrt{n})$-memory non-deterministic TM

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Same lower bounds also hold for **LCWIS**
(Longest Common **Weakly** Increasing Subsequence)

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  - For $|\Sigma| = 3$ we can, in linear time! [Duraj '13]
  - What about $|\Sigma| > 3$?

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$(1+\varepsilon)$-approximation of LCIS in $O(n^{1.5}\mathrm{polylog}(n))$ time

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Thank you!