Monochromatic Triangles, Intermediate Matrix Products, and Convolutions

Andrea Lincoln[†] **Adam Polak**[‡] Virginia Vassilevska Williams[†]

[†] Massachusetts Institute of Technology

[‡] Jagiellonain University in Kraków

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$$C_{ij} = \sum_k A_{ik} \cdot B_{kj}$$

Product of two $n \times n$ matrices in time:

$O(n^3)$		naïve
$O(n^{2.81})$	1969	Strassen
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$O(n^{2.48})$	1986	Strassen
$O(n^{2.376})$	1987	Coppersmith, Winograd
$O(n^{2.373})$	2010	Stothers
$O(n^{2.3729})$	2012	Vassilevska Williams
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Big open question:
Does $\omega = 2$?

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APSP-Hypothesis: no $O(n^{2.99})$ algorithm

Easy

$(+, \times)$-product

$$O(n^{\omega})$$
$$\omega < 2.373$$

Hard

$(\min, +)$-product

$$O(n^{3-o(1)})$$

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Equality product $(+, =)$

Dominance product $(+, \leqslant)$

Minimum witness product

All-Pairs LCA in DAGs

$(\min, \max)$-product

All-Pairs Bottleneck Paths

Approximate APSP

$(\min, \leqslant)$-product

All-Pairs Increasing Paths

Unweighted directed APSP

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Wide open question: Is the same running time a coincidence?

Convolution World

Intermediate

Dominance convolution

Apx pattern matching

Min-witness convolution

$(\min, \max)$-convolution

Apx $(\min, +)$-convolution

Unweighted knapsack

$$\widetilde{O}(n^{3/2})$$

Easy

$(+, \times)$-convolution

$c_i = \sum_j a_j \cdot b_{i-j}$

$$O(n \log n)$$

Hard

$(\min, +)$-convolution

$c_i = \min_j a_j + b_{i-j}$

$$O(n^{2-o(1)})$$

Wide open question: Is the same running time a coincidence?

All-Edge Monochromatic Triangle, AE-MonoΔ

In: $n \times n$ matrix C, $C_{ij} =$ color of edge $i \to j$
Out: $n \times n$ matrix Δ, $\Delta_{ij} = \exists_k : C_{ij} = C_{jk} = C_{ki}$

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All-Edge Sparse Triangle, AE-SparseΔ

In: edge set E, $m = |E|$

Out: $\Delta : E \to \{0, 1\}$, $\Delta(i,j) = \exists_k : (i,j),(j,k),(k,i) \in E$

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Equivalent to *listing* up to m triangles [DKPVW'19]

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AE-SparseΔ in $m^{\frac{2\omega}{\omega+1}}$ time [AYZ'97]

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AE-SparseΔ in $m^{\frac{2\omega}{\omega+1}-\varepsilon}$ gives AE-MonoΔ in $n^{\frac{3+\omega}{2}-\delta}$ [VWWY'10]

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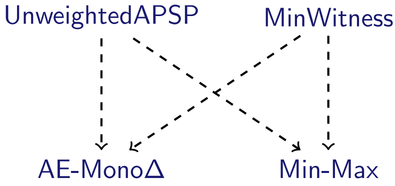
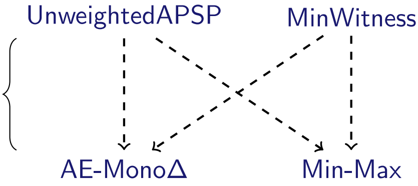
AE-MonoΔ in $n^{5/2-\varepsilon}$ gives 3SUM in $n^{2-\delta}$

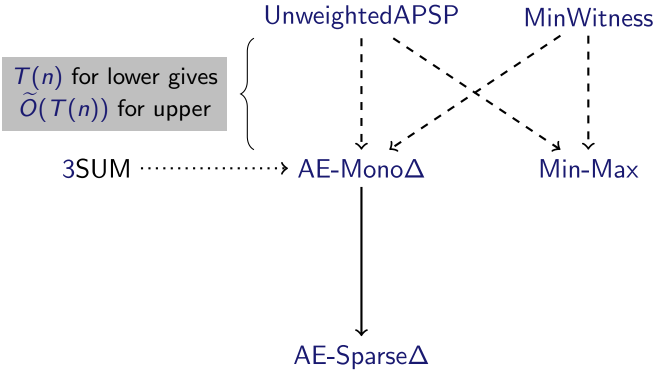
In: $a_1, a_2, \ldots, a_n \in \mathbb{Z}$
Out: $a_i + a_j + a_k = 0$

AE-MonoΔ


UnweightedAPSP
MinWitness
AE-MonoΔ
Min-Max

$T(n)$ for lower gives
$\widetilde{O}(T(n))$ for upper

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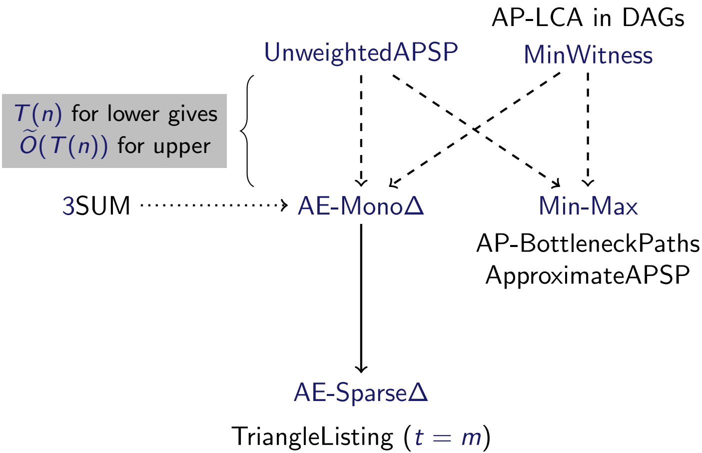
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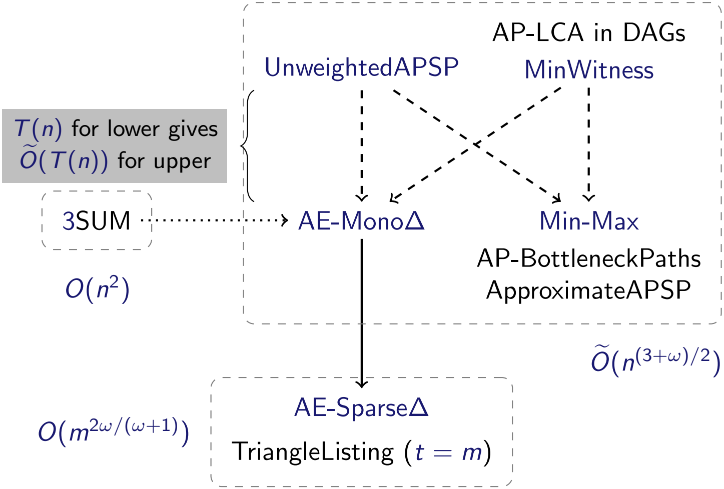
TriangleListing ($t = m$)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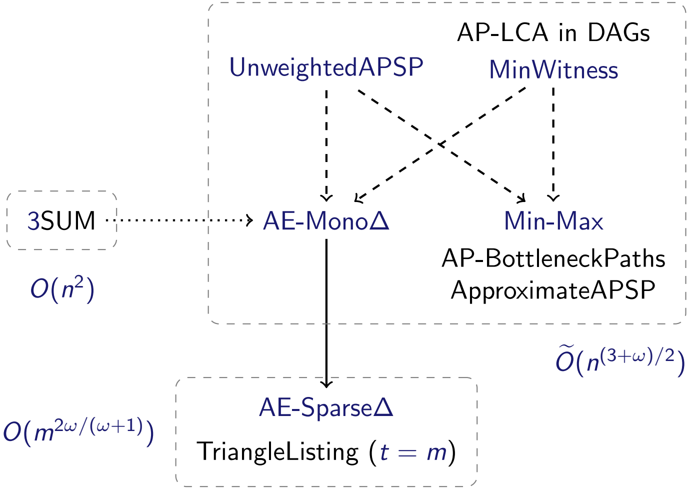
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In: Integer sequences a, b, c of length n

Out: Binary sequence d, s.t. $d[i] = \exists_j : a[j] = b[i-j] = c[i]$

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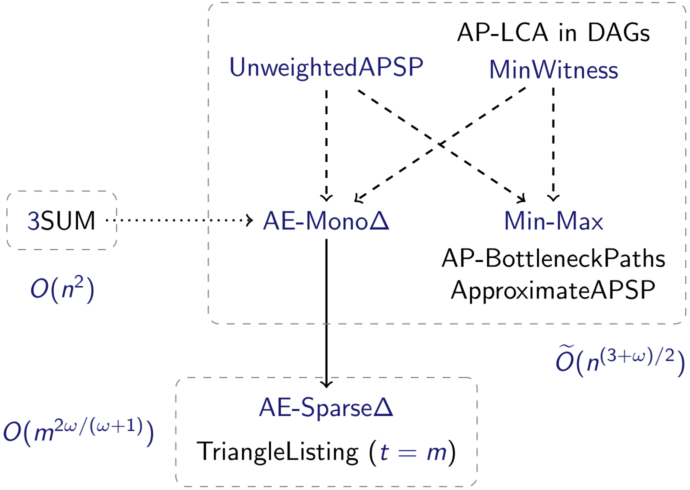
Fine-grained **equivalence** between problems of **different** runtimes!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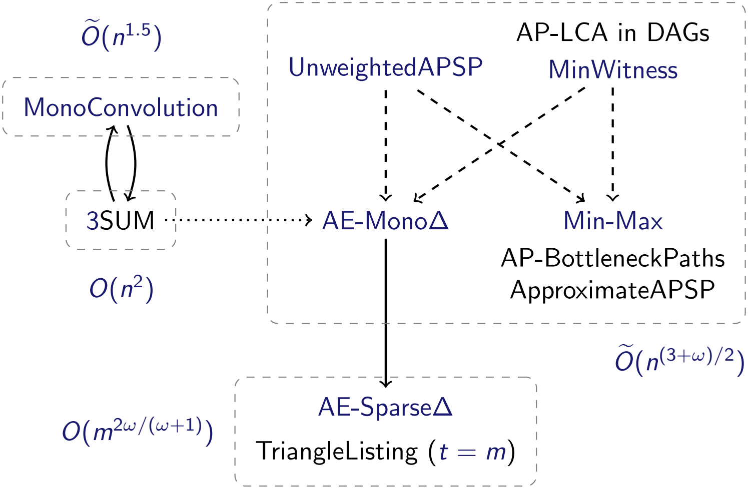
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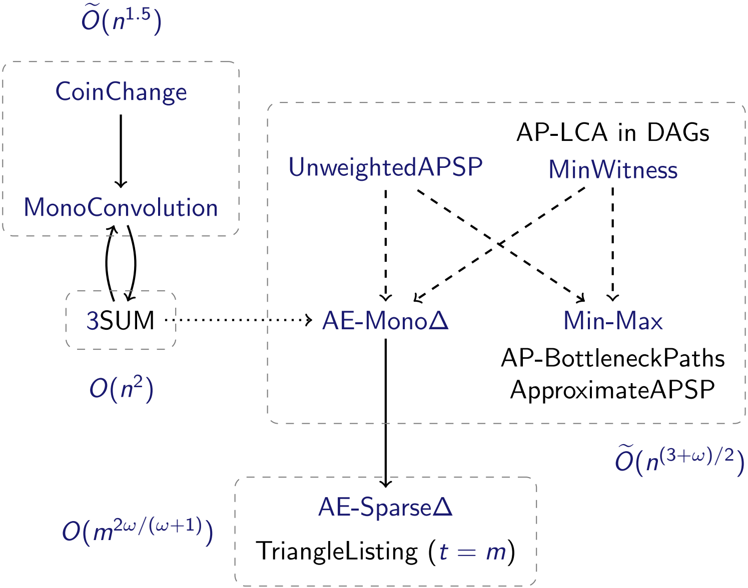
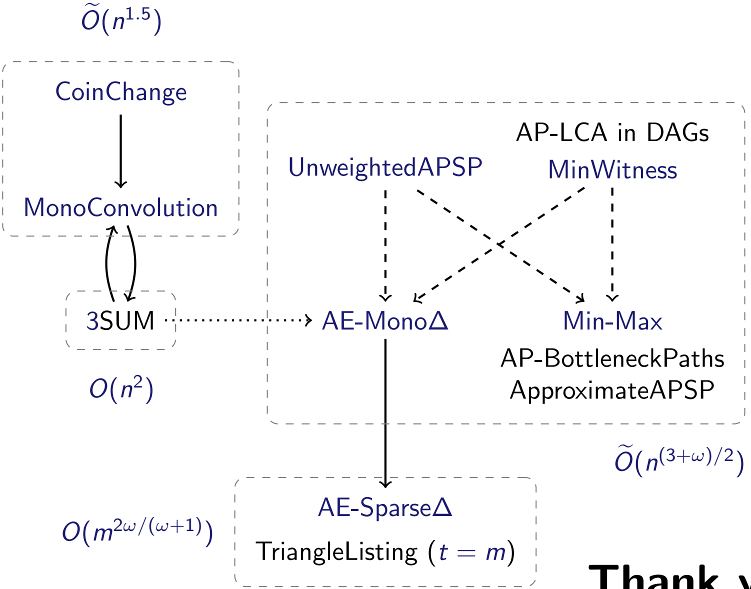

$\widetilde{O}(n^{1.5})$
CoinChange
MonoConvolution
3SUM
$O(n^2)$
AP-LCA in DAGs
UnweightedAPSP
MinWitness
AE-MonoΔ
Min-Max
AP-BottleneckPaths
ApproximateAPSP
$\widetilde{O}(n^{(3+\omega)/2})$
$O(m^{2\omega/(\omega+1)})$
AE-SparseΔ
TriangleListing ($t = m$)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Thank you!