



Homework

- In order to get a passing grade, you need to solve at least two problems.
- Please send your solutions to adam.polak@unibocconi.it no later than on May 15th, 2026. Please put “[PhD Open]” in the subject line.
- If you use consult anything or anyone while solving the problems, you should disclose it. Be as precise as possible in your citations (e.g., specific page numbers in a book, or a link to a webpage).

Problem 1. In the k -SUM problem, the input is a set of n integers, and the goal is decide if it is possible to choose exactly k out of these integers such that they sum up to 0. Give an $O(n^{\lceil k/2 \rceil})$ -time algorithm for k -SUM.

Problem 2. Given two sets of d -dimensional real-valued vectors $U, V \subseteq \mathbb{R}^d$, both of the same size $|U| = |V| = n$, the Closest Pair problem asks to compute

$$\min\{\|u - v\|_2 \mid u \in U, v \in V\},$$

where “ $\|\cdot\|_2$ ” denotes the Euclidean norm, i.e., $\|x\|_2 := \sqrt{\sum_{i=1}^d x_i^2}$. Prove that there is no $O(n^{2-\varepsilon} \text{poly}(d))$ -time algorithm for Closest Pair, for any $\varepsilon > 0$, unless SETH fails.

Problem 3. Recall two common ways to define the Orthogonal Vectors problem:

- In the **Monochromatic Orthogonal Vectors** problem, we are given a set of d -dimensional 0-1 vectors $U \subseteq \{0, 1\}^d$, of size $|U| = n$, and the goal is to decide if there exists a pair of vectors $u, v \in U$ such that $u \cdot v = 0$.
- In the **Bichromatic Orthogonal Vectors** problem, we are given two sets of d -dimensional 0-1 vectors $U, V \subseteq \{0, 1\}^d$, both of the same size $|U| = |V| = n$, and the goal is to decide if there exists a pair of vectors $u \in U$ and $v \in V$ such that $u \cdot v = 0$.

Prove that Monochromatic Orthogonal Vectors and Bichromatic Orthogonal Vectors are *subquadratic equivalent*, i.e., if there exists a truly subquadratic $O(n^{2-\varepsilon} \text{poly}(d))$ -time algorithm for either one of them, for $\varepsilon > 0$, then there also exists a truly subquadratic $O(n^{2-\varepsilon'} \text{poly}(d))$ -time algorithm for the other, for $\varepsilon' > 0$.

Problem 4. In the X+Y problem we are given two sets X and Y , each containing n integers, and we need to determine whether the sumset $X + Y = \{a + b \mid a \in X, b \in Y\}$ contains n^2 distinct integers. Prove that if the X+Y problem can be solved in truly subquadratic time, then 3SUM can also be solved in truly subquadratic time.

Problem 5. In the Maximum Submatrix problem we are given an n -by- n integer matrix $A \in \mathbb{Z}^{n \times n}$, and the goal is to find a rectangular submatrix with the largest sum of entries. In other words, we are looking for r_1, r_2, c_1, c_2 satisfying $1 \leq r_1 \leq r_2 \leq n$, $1 \leq c_1 \leq c_2 \leq n$ that maximize $\sum_{r=r_1}^{r_2} \sum_{c=c_1}^{c_2} A[r, c]$. Prove that there is no $O(n^{3-\varepsilon})$ -time algorithm for Maximum Submatrix, for any $\varepsilon > 0$, unless the APSP Hypothesis fails.